Inference on Risk Premia in the Presence of Omitted Factors

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Theoretical asset pricing models predict that some factors are priced.

Such models are typically very stylized, with 1-2 factors, e.g., CAPM, hence rejected by the data.

Plausible that the literal model is missing some factors (Ross’ APT) or the factor of interest is measured with error (Roll’s critique).

Can we address whether the factor predicted by a theoretical model is priced, without specifying all factors in the model, while allowing for measurement error?
Introduction

- Focus on the **risk premium** of a factor, instead of testing the entire model
  - *How much are investors willing to pay to hedge that risk (and just that risk)?*

- Easy to do for **traded** factors
  - avg excess return of the portfolio
  - no need to specify a full-fledged model

- **Nontraded** factors: several ways

  1. Multidimensional portfolio sorting
  2. Two-pass Fama-MacBeth (FM), GMM
Introduction

Either method is biased when there are omitted factors, or the factor of interest is measured with error.

- Portfolios with high market betas have smaller excess returns than those with low market betas.
- FM estimate of market risk premium is negative and significant ≠ the average market return.
- Controlling Fama-French factors does not solve this problem.
Consider a one factor model (e.g., CAPM):

\[ r_t = \beta \gamma + \beta v_t + u_t, \quad g_t = v_t + z_t, \quad \mathbb{E}(z_t|v_t) = 0 \]

If using the standard FM regression with \( g_t \):

- \( \hat{\beta} \) is biased towards 0 (attenuation bias)
- \( \hat{\gamma} \) is biased upwards
- the smaller the signal-to-noise ratio, the worse the bias.
Consider a two factor model:

\[ r_t = \beta_1 \gamma_1 + \beta_2 \gamma_2 + \beta_1 v_{1t} + \beta_2 v_{2t} + u_t. \]

How to estimate \( \gamma_1 \) if we only observe \( g_t = v_{1t} \) and miss \( v_{2t} \)?

- even if \( v_{1t} \perp v_{2t} \), it is not enough to consistently estimate \( \gamma_1 \)
- orthogonality between factors is enough to guarantee consistent estimate of \( \beta_1 \)
- recovery of \( \gamma_1 \) requires an additional constraint \( \beta_1 \perp \beta_2 \)!

\[ \mathbb{E}(r_t) = \beta_1 \gamma_1 + \beta_2 \gamma_2 \]
This paper proposes a **three-pass** methodology to address the omitted factor and measurement error problems in the two-pass FM regressions.

- **Rotation invariance result**
  - Risk premia estimate for $g_t$ is the same for any rotation of the controls as long as they together span the entire factor space.
  - No need to know the **identities** of the omitted factors.

- **Recover the factor space via PCA from a **large** panel of returns**
  - Say something about the **economically interpretable** $g_t$, using latent factors from returns only as *controls*.
  - Exploit the benefits of factor analysis but address its well-known uninterpretability (APT).
Finance Literature

- Model misspecification and weak identification
  - In particular, there is a weak factor $v_{2t}$ included, for which $\beta_2 \approx 0$.

  \[ r_t = \beta_1 \gamma_1 + \beta_2 \gamma_2 + \beta_1 v_{1t} + \beta_2 v_{2t} + u_t. \]

  $\hat{\beta}_2$ would be close to 0, which would then mess up $\gamma_1$.

  - In contrast, we mainly focus on the case where $v_{2t}$ should be added ($\beta_2 \neq 0$) but is missing. (NEW)

- Other related work mainly focus on mispricing, i.e., “$\alpha$”
Econometrics Literature

- (Approximate) Factor model
  - Ross (1976), Chamberlain and Rothchild (1983)

- Determine the number of factors
  - Bai and Ng (2002), Onatski (2010), Ahn and Horenstein (2013)

- Inference, Forecasting, etc
Model Setup

- True model is linear with $p$ factors $v_t$ (zero-mean).

\[ r_t = \alpha + \iota_n \gamma_0 + \beta \gamma + \beta v_t + u_t \]

- $\gamma$ are the factor risk premia
- $\beta$ are the risk exposures
- Factor of interest $g_t$ (tradable or nontradable):

\[ g_t = \xi + \eta v_t + z_t \]

- $z_t$ is measurement error, that we will also account for

**Risk premium** of $g_t$ is the expected excess return of a “pure factor” portfolio with beta of 1 with $g_t$ (free of measurement error) and 0 with all other risk sources. For $g_t$, it is $\eta \gamma$. 
Rotation invariance: an example

- Standard FM using the 25 FF portfolios

<table>
<thead>
<tr>
<th>Slope</th>
<th>RmRf</th>
<th>SMB</th>
<th>HML</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>-112</td>
<td>12</td>
<td>40</td>
</tr>
<tr>
<td>Stderr</td>
<td>(44)</td>
<td>(10)</td>
<td>(11)</td>
</tr>
</tbody>
</table>

- Construct two new factors:

\[
F_2 = a \times RmRf + b \times SMB + c \times HML
\]

\[
F_3 = RmRf + 0.001HML
\]

- What happens if we run FM using \( RmRf, F_2, F_3 \)?

  - All factors are contaminated by \( RmRf \): \textbf{betas} will be wrong
  - \( a, b, c \) are unknown constants
  - \( F_3 \) is almost collinear with \( RmRf \)
Rotation invariance: an example

- Standard Fama-MacBeth using the 25 FF portfolios

<table>
<thead>
<tr>
<th></th>
<th>RmRf</th>
<th>SMB</th>
<th>HML</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>-112</td>
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</tr>
<tr>
<td>Stderr</td>
<td>(44)</td>
<td>(10)</td>
<td>(11)</td>
</tr>
</tbody>
</table>

- Rotated model:

<table>
<thead>
<tr>
<th></th>
<th>RmRf</th>
<th>$F_2$</th>
<th>$F_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>-112</td>
<td>-100</td>
<td>-1116</td>
</tr>
<tr>
<td>Stderr</td>
<td>(44)</td>
<td>(47)</td>
<td>(444)</td>
</tr>
</tbody>
</table>

- The slope of $g_t$ in any FM regression is $\eta \gamma$, as long as $g_t$, together with any linear combinations of $\nu_t$, span the true factor space.

- Multicolinearity in these regressions is not a problem.
Rotation Invariance Result

In the case when $v_t$ is latent and $g_t$ is measured with error (focus of this paper): it is natural to use PCA to recover the factor space, and use an additional regression on latent factors to de-noise

$$r_t = \alpha + \iota_n \gamma_0 + \beta H^{-1} H \gamma + \beta H^{-1} H v_t + u_t$$

$$g_t = \xi + \eta H^{-1} H v_t + z_t$$

The risk premium of $g_t$ in the rotated model is $\eta H^{-1} H \gamma = \eta \gamma$, regardless of the rotation $H$.

- Risk exposure ($\beta$) is not rotation-invariant.
A Three-pass Estimator: Invariance Result + PCA

We propose a **three-pass estimator** to obtain $\eta \gamma$.

1. Extract latent factors $\hat{v}_t$ via PC

2. Use cross-sectional regression to estimate latent factor risk premia $\hat{\gamma}$

3. Regress $g_t$ on the latent factors $\hat{v}_t$ via time-series regression to obtain $\hat{\eta}$

**Risk premium of $g_t$ is estimated as $\hat{\eta}\hat{\gamma}$**
Another View

Risk premium of $g_t$ is $-\gamma_0 \text{Cov}(g_t, m_t)$, where $m$ is the SDF

- Steps 1 and 2 recover the SDF (under APT):

$$m_t = \gamma_0^{-1}(1 - \gamma^T \Sigma_{v}^{-1} v_t)$$

- Step 3 computes risk premia as $-\gamma_0 \text{Cov}(g_t, m_t)$

Invariance holds because SDF is rotation-invariant and the risk premium depends on a univariate covariance
Risk Premia vs Risk Prices

1. What is the **risk premium** of $g_t$?
   - *How much are investors willing to pay to hedge that risk?*

2. What is the **risk price** of $g_t$?
   - *Does $g_t$ help price the cross-section of assets?*

3. They are related:

   $$\Sigma_v^{-1} \gamma = \lambda$$

   $$m_t = \gamma_0^{-1}(1 - \gamma^T \Sigma_v^{-1} v_t) = \gamma_0^{-1}(1 - \lambda^T v_t)$$

   - The invariance result does not hold for risk prices
   - Risk prices depend on the covariance matrix of the other factors $\Sigma_v$
Asymptotic Theory

- Estimator is consistent and satisfies CLT as $T, n \to \infty$. Convergence rate is $\min(n^{1/2}, T^{1/2})$.
- Derive asymptotic distribution under fairly general assumptions.
- Heteroscedasticity robust standard errors.
- The usual GLS is not necessarily more efficient than OLS so that we can avoid large-dim covariance estimates ($\Sigma^u$) in our inference, because it appears at $O_p(n^{-1/2}T^{-1/2}) << O_p(n^{-1/2} + T^{-1/2})$.
- Robust even if a few extra PCs are used.
- Also provide inference for $\hat{g}_t$, which requires $\hat{\Sigma}^u$.
- Also provide a Wald test of $H_0 : \eta = 0$ (i.e., $g_t$ is weak).
A Critical Assumption

- All latent factors are pervasive (strong):

\[ \| n^{-1} \beta^T \beta - \Sigma \beta \| = o(1), \quad \text{as} \quad n \to \infty \]

- Our view of the weak factor literature:
  The true (latent) fundamental economic factors are strong, but we do not have good (observable) proxies for such factors.

- For example, macro factors in the literature are weak, see Gospodinov et al. (2014b), Bryzgalova (2015), but stock market strongly reacts to Fed and Government policies, see Bernanke and Kuttner (2005), Pastor and Veronesi (2012, 2014).
Simulation: Estimation

Fama-MacBeth: $\gamma_0$

Fama-MacBeth: $R_m R_f$

Fama-MacBeth: SMB

Fama-MacBeth: HML

Fama-MacBeth: IP

Three-Pass: $\gamma_0$

Three-Pass: $R_m R_f$

Three-Pass: SMB

Three-Pass: HML

Three-Pass: IP
$H_0 : \eta = 0$, i.e., factor $g_t$ is weak
Testing portfolios and their factor structure

- Test assets: 202 standard equity portfolio from Fama and French
  - 25 portfolios sorted by size and book-to-market
  - 17 industry portfolios
  - 25 portfolios sorted by operating profitability and investment
  - 25 portfolios sorted by size and variance
  - 35 portfolios sorted by size and net issuance
  - 25 portfolios sorted by size and accruals
  - 25 portfolios sorted by size and momentum
  - 25 portfolios sorted by size and beta.

- We first need to select the number of PCs
  - 4 PCs have average time series $R^2$ of 65%
  - Robustness to increasing the number of PCs (4,5,6)
Portfolios vs. individual assets

- We use **portfolios** throughout, as our theory assumes constant betas
  - Individual asset betas are unstable
  - If betas are a function of characteristics, characteristics-sorted portfolios have constant betas

For instance, if $\tilde{r}_{t+1}$ is excess-return vector of individual stocks with time-varying $\beta_t$ explained by observable characteristics:

$$\tilde{r}_{t+1} = \beta_t \gamma + \beta_t v_{t+1} + \tilde{u}_{t+1}, \quad \text{and} \quad \beta_t = c_t \beta$$

then

$$r_{t+1} := (c_t^T c_t)^{-1} c_t^T \tilde{r}_{t+1} = \beta \gamma + \beta v_{t+1} + (c_t^T c_t)^{-1} \tilde{u}_{t+1}.$$
Factor selection

![Graph showing eigenvalues 1 to 8 and eigenvalues 2 to 8](image-url)
PC fit: BE/BE-ME
PC fit: Industries
PC fit: ME/Momentum

![Scatter plot showing predicted vs. realized returns]
Observable factors

- Consider many tradable and nontradable factors
  - CAPM: Rm
  - Fama-French 3-factor model (FF3): Rm, SMB, HML
  - Carhart 4-factor model (FF4): Rm, SMB, HML, Momentum
  - Fama-French 5-factor model (FF5): Rm, SMB, HML, CMA (investment), RMW (profitability)
  - Q-factor model (HXZ): Rm, ME (size), IA (investment), ROE (profitability)
  - Betting against beta (BAB)
  - Quality-minus-junk (QMJ)
  - Industrial production growth (IP)
  - Jurado, Ludvigson and Ng macro factors (LN)
  - Liquidity factor from Pastor and Stambaugh 2003 (LIQ)
  - Intermediary factors from He et al. (2016) and Adrian et al. (2015)
Results: Tradable factors, Fama-MacBeth vs. 3-pass

<table>
<thead>
<tr>
<th>Model</th>
<th>Factors</th>
<th>Avg ret</th>
<th>FM $\gamma$</th>
<th>stderr</th>
<th>3-pass, $P = 4$ $\gamma$</th>
<th>stderr</th>
<th>$R_g^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF3</td>
<td>RmRf</td>
<td>0.50</td>
<td>$-0.57$ **</td>
<td>(0.25)</td>
<td>0.37 *</td>
<td>(0.20)</td>
<td>98.18</td>
</tr>
<tr>
<td>SMB</td>
<td>0.23</td>
<td>0.17</td>
<td>(0.13)</td>
<td></td>
<td>0.23 *</td>
<td>(0.13)</td>
<td>93.90</td>
</tr>
<tr>
<td>HML</td>
<td>0.34</td>
<td>0.23</td>
<td>* (0.13)</td>
<td></td>
<td>0.21 *</td>
<td>(0.11)</td>
<td>66.86</td>
</tr>
</tbody>
</table>

- Robust to using $P=4$, 5, 6
- These are **tradable** factors, and they are not weak.
- If the model is correctly specified, we would expect the risk premia to be equal to the average excess returns
## Results: Nontradable factors

<table>
<thead>
<tr>
<th>Factors</th>
<th>FM</th>
<th>3-pass, $P = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma$</td>
<td>stderr</td>
</tr>
<tr>
<td>IP</td>
<td>$-0.13$</td>
<td>$(0.07)$</td>
</tr>
<tr>
<td>LN 1</td>
<td>$70.25$</td>
<td><strong>$(21.62)$</strong></td>
</tr>
<tr>
<td>LN 2</td>
<td>$3.84$</td>
<td>$(24.02)$</td>
</tr>
<tr>
<td>LN 3</td>
<td>$-1.71$</td>
<td>$(15.04)$</td>
</tr>
<tr>
<td>Liquidity</td>
<td>$0.02$</td>
<td>$(0.97)$</td>
</tr>
<tr>
<td>He et al.</td>
<td>$0.02$</td>
<td>$(0.64)$</td>
</tr>
<tr>
<td>Adrian et al.</td>
<td>$1.25$</td>
<td><strong>$(0.32)$</strong></td>
</tr>
</tbody>
</table>
## Factor Loadings

<table>
<thead>
<tr>
<th>Model</th>
<th>Factors</th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Factor 3</th>
<th>Factor 4</th>
<th>Factor 5</th>
<th>Factor 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPM</td>
<td>RmRf</td>
<td>91.0</td>
<td>6.3</td>
<td>1.7</td>
<td>0.1</td>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>FF3</td>
<td>SMB</td>
<td>31.0</td>
<td>64.0</td>
<td>0.6</td>
<td>0.9</td>
<td>1.0</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>HML</td>
<td>7.0</td>
<td>1.3</td>
<td>75.5</td>
<td>4.9</td>
<td>1.4</td>
<td>9.9</td>
</tr>
<tr>
<td>IP</td>
<td>IP Growth</td>
<td>2.2</td>
<td>0.5</td>
<td>4.4</td>
<td>1.0</td>
<td>3.8</td>
<td>88.0</td>
</tr>
<tr>
<td>Liq.</td>
<td>Liquidity</td>
<td>95.0</td>
<td>2.8</td>
<td>1.8</td>
<td>0.2</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>Interm.</td>
<td>He et al.</td>
<td>81.2</td>
<td>12.5</td>
<td>0.1</td>
<td>2.9</td>
<td>3.2</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>Adrian et al.</td>
<td>20.3</td>
<td>6.8</td>
<td>52.0</td>
<td>16.4</td>
<td>0.0</td>
<td>4.5</td>
</tr>
</tbody>
</table>
Measurement Error

![Model: rmrf, factor #1](image1)

![IP growth](image2)
Robustness to Choice of Testing Portfolios
Robustness Across Time Periods

![Graphs showing various statistical distributions and empirical data.](image-url)

- **Intermediary (He et al.)**
- **Intermediary (Adrian et al.)**
- **Ip Growth**
- **Liquidity**

*Figures and data visualizations illustrating the robustness across different time periods.
# Risk Premia across Asset Classes

<table>
<thead>
<tr>
<th>Model</th>
<th>Factors</th>
<th>202 equity</th>
<th>FF25, 100 non-eq.</th>
<th>100 non-equity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\gamma$</td>
<td>stderr</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>FF3</td>
<td>RmRf</td>
<td>0.29</td>
<td>(0.25)</td>
<td>0.73***</td>
</tr>
<tr>
<td></td>
<td>SMB</td>
<td>0.18</td>
<td>(0.14)</td>
<td>0.22*</td>
</tr>
<tr>
<td></td>
<td>HML</td>
<td>0.25*</td>
<td>(0.13)</td>
<td>0.16**</td>
</tr>
<tr>
<td>IP</td>
<td>IP</td>
<td>−0.00</td>
<td>(0.01)</td>
<td>0.01</td>
</tr>
<tr>
<td>Liq.</td>
<td>Liq.</td>
<td>0.22</td>
<td>(0.14)</td>
<td>0.24</td>
</tr>
<tr>
<td>Interm.</td>
<td>He</td>
<td>0.28</td>
<td>(0.30)</td>
<td>1.03***</td>
</tr>
<tr>
<td></td>
<td>Adrian</td>
<td>0.77***</td>
<td>(0.17)</td>
<td>0.50***</td>
</tr>
</tbody>
</table>
Conclusion

- Marry PCA with invariance result to correct for omitted variable bias

- Two caveats:
  1. PCA might miss latent weak factors
     - Tradable factor results
     - Robustness to the number of factors
     - Good deal bounds
  2. Invariance result does not hold for the risk price of $g_t$
     - Use tools from machine learning/model selection to recover the omitted factors and control for them (Feng, Giglio and Xiu 2017).

- Main point: when computing risk premia, literature typically use few arbitrary factors as controls. Crucial to complete the space.