Supplement to “Resolution of Policy Uncertainty and Sudden Declines in Volatility”

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Abstract
This appendix contains details about variance swap pricing, invariant transformations, extended canonical forms, volatility models, likelihood estimation, as well as additional tables and figures.

A Variance Swap Pricing

Proof of Proposition 1. Recall that since \(X\) is affine, the generalized conditional characteristic function (GCCF) of \(X_s\) is defined below, for any \(s \geq t\) with \(t\) fixed:

\[
\Psi(s, t, u, X_t) = \mathbb{E}_t^Q \left[ e^{u^T X_s} \right],
\]
where \(u \in \mathbb{C}^N\). There exists a closed-form formula for the GCCF function given by Duffie et al. (2000):

\[
\log \left( \Psi(s, t, u, X_t) \right) = A(s - t, u) + B(s - t, u)^T X_t,
\]

where \(A\) and \(B\) satisfy the following ordinary differential equations (ODEs):

\[
\dot{B} = (K^Q)^T B + \frac{1}{2} \sum_{i=1}^{m} (\Sigma^T B_i)^2 \beta_i + l_1 \phi(B),
\]

\[
\dot{A} = (\Lambda^Q)^T B + \frac{1}{2} \sum_{i=1}^{m} (\Sigma^T B_i)^2 \alpha_i + l_0 \phi(B),
\]
where \( B(t) = u, A(t) = 0 \), and for any \( h \in \mathbb{C}^N \),
\[
\phi(h) = \int_{\mathbb{R}^N} (e^{h^T z} - 1 - h^T z) \nu^Q(dz).
\]

Under our risk neutral specification, we have
\[
\mathbb{E}^Q \left\{ \int_t^{t+\tau} \sigma^2_s ds + \int_t^{t+\tau} \int_{\mathbb{R}} j^2 \nu^Q(dj) \right\} =: \mathbb{E}^Q \left\{ \int_t^{t+\tau} f^Q(X_s) ds \right\},
\]
where \( f^Q(X_s) = \Pi_0^Q + \Pi_1^Q X + X^\top \Pi_2 X + \exp \{ \Pi_3 + \Pi_4^\top X \} \), \( \Pi_0^Q = \Pi_0 + l_0 \int_{\mathbb{R}} j^2 \nu^Q(dj) \), \( \Pi_1^Q = \Pi_1 + l_1 \int_{\mathbb{R}} j^2 \nu^Q(dj) \), and \( \nu^Q(dj) \) is the marginal distribution of jumps in \( Y \).

Denote the transition density of the process \( X \) as \( p(X_s|s-t,X_t) \), and let \( u = -iv \) in \( \Psi \) with \( v \in \mathbb{R}^N \), we have
\[
\mathbb{E}^Q \left( f^Q(X_s)|X_t = x \right) = \int_{\mathbb{R}^N} f^Q(x') p(x'|s-t,x) dx'
= \frac{1}{(2\pi)^N} \int_{\mathbb{R}^N} \int_{\mathbb{R}^N} f^Q(x') e^{iv \top x'} \Psi(s,t,-iv,x) dx' dv.
\]

We then utilize Fourier Transform of the tempered distributions to simplify the integral with respect to \( x' \). Consider the quadratic part first. Note that
\[
\int_{\mathbb{R}^N} (\Pi_0^Q (\Pi_1^Q)^\top x + x^\top (\Pi_2) x) e^{iv \top x} dx' = (2\pi)^N (\Pi_0^Q - i(\Pi_1^Q)^\top \nabla_v - \nabla_v (\Pi_2) \nabla_v) \delta(v),
\]
where \( \delta(\cdot) \) is a Dirac delta that satisfies \( \int_{\mathbb{R}^N} \delta(v) dv = 1 \), and \( \int_{\mathbb{R}^N} \delta(v) g(v) dv = g(0) \) for any test function \( g \). Therefore, by direct calculations we obtain
\[
\mathbb{E}^Q \left( \Pi_0^Q (\Pi_1^Q)^\top X_s + X_s^\top (\Pi_2) X_s | X_t = x \right) = \int_{\mathbb{R}^N} (\Pi_0^Q - i(\Pi_1^Q)^\top \nabla_v - \nabla_v (\Pi_2) \nabla_v) \delta(v) \Psi(s,t,-iv,x) dv
= \Pi_0^Q + (\Pi_1^Q)^\top \nabla_u \Psi(s,t,u,x) \big|_{u=0} + \nabla_u \Pi_2 \nabla_u \Psi(s,t,u,x) \big|_{u=0}.
\]

For the exponential part, similarly we have
\[
\int_{\mathbb{R}^N} e^{\Pi_3 + (\Pi_4)^\top x'} e^{iv \top x'} dx' = (2\pi)^N e^{\Pi_3} \delta(v - i\Pi_4),
\]
so that we can derive
\[
\mathbb{E}^Q \left( e^{\Pi_3 + (\Pi_4)^\top X_s} | X_t = x \right) = \int_{\mathbb{R}^N} e^{\Pi_3} \delta(v - i\Pi_4) \Psi(s,t,-iv,x) dv = e^{\Pi_3} \Psi(s,t,\Pi_4,x).
\]
The pricing formula for variance swaps follows immediately. Note that we have applied properties of tempered distributions to simplify the calculations, all of which can be found in Kanwal (2004).
B Invariant Transformations

Proof of Proposition 2. To prove the existence, we extend Dai and Singleton (2000) and Ahn et al. (2002) to provide invariant transformations of the general model. These transformations lead to alternative specifications without altering the price of variance swaps (or more generally, the likelihood of the observables). We summarize the state factors, Brownian motions, jumps, and parameter vectors in \( \theta \):

\[
\theta = \left( X_t, W_t^Q, Z_t^Q, \Lambda^Q, K^Q, \Sigma, \{ \alpha_i, \beta_i \} _{1 \leq i \leq m}, \tilde{\nu}(\cdot, dz), \Pi_0, \Pi_1, \Pi_2, \Pi_3, \Pi_4 \right).
\]

There are 4 classes of admissible transformations that ensure the transformed process to follow (1), (2), (3), and (4) of the main text (with different set of parameters), while maintaining the same observable implications:

An Affine Transformation \( \mathcal{T}_A \) refers to \( \mathcal{T}_A X_t = \mathcal{Y} + \mathcal{L} X_t \), where \( \mathcal{Y} \) is an \( N \times 1 \) vector and \( \mathcal{L} \) is an \( N \times N \) nonsingular matrix. As a result, \( \mathcal{T}_A \theta \) is defined below.

\[
\mathcal{T}_A \theta = \begin{pmatrix}
\mathcal{Y} + \mathcal{L} X_t, W_t^Q, \mathcal{L} Z_t^Q, \mathcal{L} \Lambda - \mathcal{L} K \mathcal{L}^{-1} \mathcal{Y}, \mathcal{L} K \mathcal{L}^{-1}, \mathcal{L} \Sigma,
\{ \alpha_i - \beta_i^T \mathcal{L}^{-1} \mathcal{Y}, \mathcal{L}^T \mathcal{L}^{-1} \beta_i \} _{1 \leq i \leq m}, \tilde{\nu}(\mathcal{L}^{-1} (\cdot + \mathcal{Y}), \mathcal{L} dz),
\Pi_0 - (\Pi_1)^T \mathcal{L}^{-1} \mathcal{Y} + \mathcal{Y}^T \mathcal{L}^{-1} (\Pi_2) \mathcal{L}^{-1} \mathcal{Y}, \mathcal{L}^T \mathcal{L}^{-1} \Pi_1 - 2 \mathcal{L}^T \Pi_2 \mathcal{L}^{-1} \mathcal{Y}, \mathcal{L}^T \mathcal{L}^{-1} \Pi_2 \mathcal{L}^{-1},
\Pi_3 - (\Pi_4)^T \mathcal{L}^{-1} \mathcal{Y}, \mathcal{L}^T \mathcal{L}^{-1} \Pi_4
\end{pmatrix}.
\]

An Orthonormal Rotation \( \mathcal{T}_O \) refers to an affine transformation on the Brownian factor \( W_t^Q \) such that \( \mathcal{T}_O W_t^Q = OW_t^Q \), where \( O \) is an orthonormal matrix satisfying \( O^T O = OO^T = I_{N \times N} \).

\[
\mathcal{T}_O \theta = \begin{pmatrix}
X_t, OW_t^Q, Z_t^Q, \Lambda^Q, K^Q, \Sigma O^T, \{ \alpha_i, \beta_i \} _{1 \leq i \leq m}, \tilde{\nu}(\cdot, dz), \Pi_0, \Pi_1, \Pi_2, \Pi_3, \Pi_4
\end{pmatrix}.
\]

A Diffusion Rescaling \( \mathcal{T}_D \) rescales the diagonal elements of \( S_t \) by a nonsingular diagonal matrix \( D \) in \( \mathbb{R}^{N \times N} \). That is,

\[
\mathcal{T}_D \theta = \begin{pmatrix}
X_t, W_t^Q, Z_t^Q, \Lambda^Q, K^Q, \Sigma D^{-1}, \{ D_{ii}^Q \alpha_i, D_{ii}^Q \beta_i \} _{1 \leq i \leq m}, \tilde{\nu}(\cdot, dz), \Pi_0, \Pi_1, \Pi_2, \Pi_3, \Pi_4
\end{pmatrix}.
\]

A Permutation \( \mathcal{T}_P \) alters the order of state variables, which has no observable effect.

Using these transformations, we can impose normalizations on the process to achieve its canonical representation. Following exactly the same procedure as in Appendix C of Dai and Singleton (2000), we normalize the parameters in the dynamics of (2), while leaving the parameters in (3) unrestricted (barring from their positivity constraints). Once we have transformed any model of (2) into its canonical form, no restrictions can be imposed on the parameters in (3) without affecting this canonical form, so that the procedure achieves the maximal model.

To show the uniqueness, by the existence result, it is equivalent to prove that canonical forms of different types are not observationally equivalent under these invariant transformations. This is obvious from Dai and Singleton (2000), because the number of positive factors remains unchanged under admissible transformations. ❑
C Extended Canonical Forms

Here we provide canonical forms that allow for pure jump volatility factors. Each model of this class is assigned to a family $\mathbb{A}_{m,j}(N,J)$, in which $N$ is the number of Brownian state variables, $J$ is the number of pure jump factors, while $m$ and $j$ are the number of independent linear combinations of those state variables that are positive, respectively. In the absence of pure jump factors, we recycle the notation $\mathbb{A}_{m}(N)$ in Dai and Singleton (2000), and provide the canonical forms in the main text.

For each $m$ and $j$, we partition $X^T = (X_{m \times 1}^T, X_{j \times 1}^T, X_{(N-m) \times 1}^T, X_{(J-j) \times 1}^T)^T$. The canonical representation takes a special form of equation (3) in the main text, where for $m > 0$,

$$K_Q = \begin{pmatrix}
K_{m \times m}^Q & K_{m \times j}^Q & 0_{m \times (N-m)} & 0_{m \times (J-j)} \\
K_{j \times m}^Q & K_{j \times j}^Q & 0_{j \times (N-m)} & 0_{j \times (J-j)} \\
K_{(N-m) \times (N-m)}^Q & K_{(N-m) \times j}^Q & K_{(N-m) \times (N-m)}^Q & K_{(N-m) \times (J-j)}^Q \\
K_{(J-j) \times m}^Q & K_{(J-j) \times j}^Q & K_{(J-j) \times (N-m)}^Q & K_{(J-j) \times (J-j)}^Q 
\end{pmatrix},$$

and $K_{(N-m) \times (N-m)}^Q$ and $K_{(J-j) \times (J-j)}^Q$ is either the upper or lower triangle for $m = 0$ or $j = 0$, respectively. In addition,

$$\Lambda_Q = \begin{pmatrix}
\Lambda_{m \times 1}^Q \\
0_{j \times 1} \\
0_{(N+J-m-j) \times 1}
\end{pmatrix}, \quad \Sigma = \begin{pmatrix}
1_{N \times N} \\
0_{J \times J}
\end{pmatrix}, \quad \alpha = \begin{pmatrix}
0_{(m+j) \times 1} \\
1_{(N-m) \times 1} \\
0_{(J-j) \times 1}
\end{pmatrix}, \quad \beta_1 = \begin{pmatrix}
\beta_{1,m \times 1} \\
\beta_{1,j \times 1} \\
0_{(N-m) \times 1} \\
0_{(J-j) \times 1}
\end{pmatrix},$$

with restrictions such that for $1 \leq i \neq k \leq m$, $(m+1) \leq s \neq t \leq m+j$ and $1 \leq j \leq N+J$,

$$K_{i,k}^Q \geq 0, \quad K_{s,t}^Q \geq 0, \quad \text{Re}(\text{Eigen}(K_Q^Q)) < 0, \quad \Lambda_i^Q - l_0 \int \bar{\nu}(z)dz \geq \frac{1}{2}, \quad \Lambda_s^Q \geq 0, \quad B_{ij} \geq 0, \quad B_{sj} \geq 0, \quad l_{1,i}^Q \geq 0, \quad l_{1,s}^Q = 1 \text{ or } 0, \quad l_0^Q \geq 0, \quad l_0^Q \Pi_{i=1}^{m} \Pi_{s=1}^{m+j} l_{1,s}^Q \neq 0, \quad \bar{\nu}(\mathbb{R}^{m+j} \times \mathbb{R}^{N+J-m-j}) = 0,$$

where $\bar{K}_Q$ is of the same size of $K_Q$, and

$$\bar{K}_{m \times m}^Q = K_{m \times m}^Q - \text{Diag} \left( l_{1,i} \int \bar{\nu}(z)dz \right)_{1 \leq i \leq m},$$

$$\bar{K}_{j \times j}^Q = K_{j \times j}^Q - \text{Diag} \left( l_{1,s} \int \bar{\nu}(z)dz \right)_{m+1 \leq s \leq m+j}.$$
D Summary of Two Factor Volatility Models

D.1 $A_0(2)$ Model

The $A_0(2)$ model specifies the dynamics of $X$ as:

$$
\begin{bmatrix}
    dX_{1t} \\
    dX_{2t}
\end{bmatrix} = \left( \begin{bmatrix} \kappa_{11}^Q & 0 \\ \kappa_{21}^Q & \kappa_{22}^Q \end{bmatrix} \begin{bmatrix} X_{1t} \\
    X_{2t} \end{bmatrix} \right) dt + \begin{bmatrix} dW_{1t}^Q \\
    dW_{2t}^Q \end{bmatrix} + \begin{bmatrix} dZ_{1t}^Q \\
    dZ_{2t}^Q \end{bmatrix}.
$$

Jumps follow compound Poisson processes with independent jump sizes following double exponential distributions:

$$
\text{size of } Z_{1t}^Q \sim \begin{cases} 
    \exp(\beta_{1+}^Q), & q_1 \\
    -\exp(\beta_{1-}^Q), & 1 - q_1
\end{cases}, \quad \text{and}
$$

$$
\text{size of } Z_{2t}^Q \sim \begin{cases} 
    \exp(\beta_{2+}^Q), & q_2 \\
    -\exp(\beta_{2-}^Q), & 1 - q_2
\end{cases}.
$$

Their intensity is specified as $l_0$.

For this model, we specify the dynamics under $P$ as

$$
\begin{bmatrix}
    dX_{1t} \\
    dX_{2t}
\end{bmatrix} = \left( \begin{bmatrix} \lambda_1^P & 0 \\ \lambda_2^P & \kappa_{21}^P & \kappa_{22}^P \end{bmatrix} \begin{bmatrix} X_{1t} \\
    X_{2t} \end{bmatrix} \right) dt + \begin{bmatrix} dW_{1t}^P \\
    dW_{2t}^P \end{bmatrix} + \begin{bmatrix} dZ_{1t}^P \\
    dZ_{2t}^P \end{bmatrix},
$$

where jumps in $Z_{1t}^P$ and $Z_{2t}^P$ are specified with the same mixture probabilities but in different sizes $\beta_{1,+/-}^P$ and $\beta_{2,+/-}^P$.

The parameter constraints in this model are given by:

$$
\kappa_{11}^Q < 0, \quad \kappa_{22}^Q < 0, \quad l_0 \geq 0, \quad \kappa_{11}^P < 0, \quad \kappa_{22}^P < 0.
$$

D.2 $A_1(2)$ Model

Another model that incorporates negative jumps can be specified as

$$
\begin{bmatrix}
    dX_{1t} \\
    dX_{2t}
\end{bmatrix} = \left( \begin{bmatrix} \lambda_1^Q & 0 \\ 0 & \kappa_{21}^Q & \kappa_{22}^Q \end{bmatrix} \begin{bmatrix} X_{1t} \\
    X_{2t} \end{bmatrix} \right) dt + \begin{bmatrix} \sqrt{X_{1t}} & 0 \\
    0 & \sqrt{1 + \beta_{21}X_{1t}} \end{bmatrix} \begin{bmatrix} dW_{1t}^Q \\
    dW_{2t}^Q \end{bmatrix} + \begin{bmatrix} dZ_{1t}^Q \\
    dZ_{2t}^Q \end{bmatrix},
$$

where $X_1$ is a square-root factor, and $X_2$ is an Ornstein-Uhlenbeck factor. Jumps of $X_1$ and $X_2$ follow compound Poisson processes with independent jump sizes satisfying the exponential or double exponential distributions:

$$
\text{size of } Z_{1t}^Q \sim \exp(\beta_{1+}^Q), \quad \text{and} \quad \text{size of } Z_{2t}^Q \sim \begin{cases} 
    \exp(\beta_{2+}^Q) & \text{with probability } q_2 \\
    -\exp(\beta_{2-}^Q) & \text{with probability } 1 - q_2
\end{cases}.
$$

Their intensity is specified as $l_0 + l_{11}X_{1t}$.
For this model, we specify the dynamics under the objective measure \( \mathbb{P} \) as
\[
\begin{bmatrix}
\frac{dX_{1t}}{dX_{2t}}
\end{bmatrix}
= \left( \begin{bmatrix}
\lambda_1^P & 0 \\
\kappa_{11}^P & \kappa_{21}^P \\
\kappa_{12}^P & \kappa_{22}^P
\end{bmatrix}
\right)
\begin{bmatrix}
X_{1t} \\
X_{2t}
\end{bmatrix}
+ \sqrt{X_{1t}} \begin{bmatrix} 0 \\ \sqrt{1 + \beta_{21}^P X_{1t}} \end{bmatrix}
\begin{bmatrix}
\frac{dW_{1t}^P}{dW_{2t}^P} \\
\frac{dZ_{1t}^P}{dZ_{2t}^P}
\end{bmatrix}.
\]
Jumps are of the same type with the same intensity and mixture probability but different sizes \( \beta_{1+}^P, \beta_{2+}^P, \) and \( \beta_{2-}^P. \)

The parameter constraints in this model are given by:
\[
\begin{align*}
\kappa_{11}^Q &< l_{11} \beta_{1+}^Q, & \kappa_{22}^Q &< 0, & \lambda_1^Q - l_0 \beta_{1+}^Q &\geq \frac{1}{2}, & \beta_{21} &\geq 0, & l_0 &\geq 0, & l_{11} &\geq 0, \\
\kappa_{11}^P &< l_{11} \beta_{1+}^P, & \kappa_{22}^P &< 0, & \lambda_1^P - l_0 \beta_{1+}^P &\geq \frac{1}{2}.
\end{align*}
\]

### D.3 \( A_2(2) \) Model

The dynamics of the state variables in the \( A_2(2) \) model is specified as
\[
\begin{bmatrix}
\frac{dX_{1t}}{dX_{2t}}
\end{bmatrix}
= \left( \begin{bmatrix}
\lambda_1^Q & 0 \\
\kappa_{11}^Q & \kappa_{21}^Q \\
\kappa_{12}^Q & \kappa_{22}^Q
\end{bmatrix}
\right)
\begin{bmatrix}
X_{1t} \\
X_{2t}
\end{bmatrix}
+ \sqrt{X_{1t}} \begin{bmatrix} 0 \\ \sqrt{X_{2t}} \end{bmatrix}
\begin{bmatrix}
\frac{dW_{1t}^Q}{dW_{2t}^Q} \\
\frac{dZ_{1t}^Q}{dZ_{2t}^Q}
\end{bmatrix}.
\]
where jumps in \( Z_{1t} \) and \( Z_{2t} \) cannot be negative. The intensity of jumps is \( l_0 + l_{11} X_{1t} + l_{12} X_{2t}. \)

The corresponding \( \mathbb{P} \) measure dynamics is specified as:
\[
\begin{bmatrix}
\frac{dX_{1t}}{dX_{2t}}
\end{bmatrix}
= \left( \begin{bmatrix}
\lambda_1^P & 0 \\
\kappa_{11}^P & \kappa_{21}^P \\
\kappa_{12}^P & \kappa_{22}^P
\end{bmatrix}
\right)
\begin{bmatrix}
X_{1t} \\
X_{2t}
\end{bmatrix}
+ \sqrt{X_{1t}} \begin{bmatrix} 0 \\ \sqrt{X_{2t}} \end{bmatrix}
\begin{bmatrix}
\frac{dW_{1t}^P}{dW_{2t}^P} \\
\frac{dZ_{1t}^P}{dZ_{2t}^P}
\end{bmatrix},
\]
with exponentially distributed jumps and different mean parameters.

The parameter constraints in this model are given by:
\[
\begin{align*}
\text{Re} \left( \text{Eigen} \left( \begin{bmatrix}
\kappa_{11}^Q - l_{11} \beta_{1+}^Q & \kappa_{12}^Q \\
\kappa_{21}^Q & \kappa_{22}^Q - l_{22} \beta_{2+}^Q
\end{bmatrix} \right) \right) &< 0, & \lambda_1^Q - l_0 \beta_{1+}^Q &\geq \frac{1}{2}, & \lambda_2^Q - l_0 \beta_{2+}^Q &\geq \frac{1}{2}, \\
\text{Re} \left( \text{Eigen} \left( \begin{bmatrix}
\kappa_{11}^P - l_{11} \beta_{1+}^P & \kappa_{12}^P \\
\kappa_{21}^P & \kappa_{22}^P - l_{22} \beta_{2+}^P
\end{bmatrix} \right) \right) &< 0, & \lambda_1^P - l_0 \beta_{1+}^P &\geq \frac{1}{2}, & \lambda_2^P - l_0 \beta_{2+}^P &\geq \frac{1}{2},
\end{align*}
\]
\[
\kappa_{12}^Q \geq 0, & \kappa_{21}^Q \geq 0, & \kappa_{12}^P \geq 0, & \kappa_{21}^P \geq 0, & l_0 &\geq 0, & l_{11} &\geq 0, & l_{12} &\geq 0.
\]

### E Likelihood Inference in Detail

Below we give a more detailed description of the Gibbs blocks used in the posterior simulator. For the purpose of concreteness in this section we focus on the \( A_1(2) \) model, which contains one Ornstein-Uhlenbeck factor and one square-root factor.
E.1 Time Discretization and Joint Likelihood

A time discretization of the model with time interval $\Delta$ yields

$$y_i := Y_i - Y_{(i-1)} = \mu \Delta + \sigma_{i-1} \sqrt{\Delta} \left[ \sqrt{1 - \rho_1^2 - \rho_2^2} \epsilon_{0i} + \rho_1 \epsilon_{1i} + \rho_2 \epsilon_{2i} \right] + j_i n_i,$$

(E.1)

$$X_{1i} - X_{1(i-1)} = \left[ \lambda_1 + \kappa_{11} X_{1(i-1)} \right] + \sqrt{X_{1(i-1)} \Delta} \epsilon_{1i} + z_{1i} n_i,$$

$$X_{2i} - X_{2(i-1)} = \left[ \lambda_2 + \kappa_{21} X_{1(i-1)} + \kappa_{22} X_{2(i-1)} \right] \Delta + \sqrt{X_{2(i-1)} \Delta} \epsilon_{2i} + z_{2i} n_i,$$

where $n_i$ denotes the jump time indicator that takes the value one if there is a jump on that day, and $\epsilon_{0i}, \epsilon_{1i},$ and $\epsilon_{2i}$ are standard normal variates with zero correlations, $j_i, z_{1i},$ and $z_{2i}$ are Gaussian, Gamma, and mixture of Gammas, respectively. Note that $\mu = \mu^p - l_0 \mu^p, \lambda_1 = \lambda_1^p - l_0 \beta_{1+}^p, \lambda_2 = \lambda_2^p - l_0 (q_2 \beta_{2+}^p - (1-q_2) \beta_{2-}^p), \kappa_{11} = \kappa_{11}^p - l_{11} \beta_{1+}^p, \kappa_{21} = \kappa_{21}^p - l_{11} (q_2 \beta_{2+}^p - (1-q_2) \beta_{2-}^p),$ and $\kappa_{22} = \kappa_{22}^p - l_{12} (q_2 \beta_{2+}^p - (1-q_2) \beta_{2-}^p)$.

The joint likelihood of the observables is then given by:

$$\mathcal{L}(Y, P | V, \Theta, \mathcal{A}_1(1)) = \prod_{i=1}^T p(y_i, X_i | X_{i-1}, j_i, z_i, n_i) \times p(j_i, z_i, n_i | X_{i-1}) \times p(P_i | X_i, \Theta),$$

which includes both the likelihood from the Euler discretization of the process and the likelihood of the variance swap rates. Eraker et al. (2003) show that discretization performs well with daily data. Alternatively, one could introduce a set of auxiliary data points in between of each pair of sampled latent variables and integrate them out of the likelihood function by MCMC.

E.2 Jump Times and Sizes

In our application the jump indicator $n_i$ is a binary random variable (taking on 0 or 1). To compute the Bernoulli probability, we use the conditional density of increments to volatility and returns to get that $\Pr(n_i = 1 | V, \Theta, Y, P),$ which is equal to

$$p(y_i, X_i | X_{(i-1)}, j_i, z_{1i}, z_{2i}, n_i = 1, \Theta) \times \Pr(n_i = 1 | X_{1(i-1)})$$

$$\sum_{s=0,1} p(y_i, X_i | X_{(i-1)}, j_i, z_{1i}, z_{2i}, n_i = s, \Theta) \times \Pr(n_i = s | X_{1(i-1)}),$$

where $p(y_i, X_i | X_{(i-1)}, j_i, z_{1i}, z_{2i}, n_i = s, \Theta)$ is trivariate normal with mean and covariance matrix that can be easily obtained from (E.1) and $\Pr(n_i = 1 | X_{1(i-1)}) = (l_0 + l_{11} X_{1(i-1)}) \Delta.$ Not surprisingly, the conditional posterior of jump times does not depend on the option prices directly since option prices do themselves not depend on the jump indicator.

To sample $j_i,$ we note from (9) in the main text that $p(j_i | y_i, X_i, X_{(i-1)}, n_i = 1)$ is proportional to

$$p(y_i | X_i, X_{(i-1)}, j_i, n_i = 1) \times p(j_i | n_i = 1).$$

Completing the square in the previous expression we can easily obtain the mean and variance for the conditional posterior of $j_i$ which is normal.
Analogous computations allow us to sample \( z_{1,i} \) and \( z_{2,i} \), which have a discrete scale mixture of truncated normals (TN) with a mixing variate that takes a positive (negative) value with mean \( \mu_{k+,i}^* \) (\( \mu_{k-,i}^* \)) that can be easily obtained for \( k = 1, 2 \) by completing the squares. That is, if \( s_{k,i} \in \{0,1\} \), with \( \Pr(s_{k,i} = 1|Y_i, X_i, \Theta) = q_k \), then

\[
z_{k,i} = s_{k,i} \cdot \text{TN}(\mu_{k+,i}^*, \sigma_{k,i}^2; z_{k,i} > 0) + (1 - s_{k,i}) \cdot \text{TN}(\mu_{k-,i}^*, \sigma_{k,i}^2; z_{k,i} < 0),
\]

where \( \sigma_{k,i}^2 \) denotes the corresponding conditional posterior variance of the jump size in \( z_{k,i} \).

Finally, when \( n_i = 0 \), the conditional posteriors of \( j_i \), \( z_{1,i} \) and \( z_{2,i} \) are the priors implied by the model assumptions, as the data provide no information about them.

### E.3 Latent Factors

The conditional posterior for latent factors is not known in closed-form. To sample from it, we collect terms in (9) where \( X_i \) is included, which is proportional to

\[
p(X_{i+1}|X_i, z_{1,i+1}, z_{2,i+1}, n_{i+1}) \times p(X_i|X_{i-1}, z_{1,i}, z_{2,i}, n_i) \\
\times p(y_{i+1}|X_{i+1}, j_{i+1}, n_{i+1}) \times p(y_i|X_{i-1}, X_i, j_i, n_i) \times p(P_i|X_i, \Theta) \times p(n_{i+1}|X_i),
\]

where the first five densities are Gaussian and the last term is binomial. At the \( g \)-th iteration of the sampler, we then draw from its conditional posterior using a random-walk Metropolis algorithm with the Gaussian proposal density with mean and variance computed as in Proposition 2 of Eraker (2001) but taking into account the presence of jumps. The acceptance rate of this step is in the 20-30% range for all models.

### E.4 \( \Theta_M \) and \( \Theta_P \)

Conditional on jump sizes, jump times, spot variance, short-term variance level, and remaining parameter vectors, the posterior of \( \Theta_M \) is proportional to (9). Since this conditional distribution is nonstandard, it is sampled using a Metropolis step with a normal source density centered at the current draw and covariance matrix proportional to the Hessian of \( \mathcal{L}(Y, P|V, \Theta, M) \cdot \mathcal{H}(V|\Theta, M) \) at the peak of \( \Theta_M \). The Hessian was computed by concentrating the latent variables and remaining parameters on their posterior means from a preliminary run of the algorithm. An analogous but simpler procedure, since \( \mathcal{H}(V|\Theta, M) \) does not appear in the conditional posterior, allows us to draw \( \Theta_P \). The acceptance rate of this step is around 20% for all three models. The priors are relatively uninformative but still impose the relevant constraints.

### E.5 \( \Theta_P \) and \( \Theta_E \)

A similar procedure to the one mentioned above can be used to sample \( \Theta_P \). In practice, however, since those parameters do not depend on variance swap rates once we condition on \( V \), it is often
the case that the conditional posterior distribution is available and therefore one can sample from it directly. The same comment applies to the variances of pricing errors as long as one chooses appropriately both the pricing error distributions and priors.

As for $\beta_{1+}$, recall $z_{1,i} \sim \text{Exponential}(\beta_{1+})$, so that conditional on $z_{1,i}$, and setting a conjugate prior for $\beta_{1+}$, say $\pi_{\beta_{1+}}(\beta_{1+}) \sim \text{InvGam}(\delta_{\beta_{1+}1}, \delta_{\beta_{1+}2})$, we have that $\beta_{1+} \mid \ldots \sim \text{invGam}(\delta^{*}_{\beta_{1+}1}, \delta^{*}_{\beta_{1+}2})$ with $\delta^{*}_{\beta_{1+}1} = N_J + \delta_{\beta_{1+}1}$ and $\delta^{*}_{\beta_{1+}2} = \delta_{\beta_{1+}2} + \sum_{i=1}^{N_J} z_{1,i}$ and where $N_J = \sum_{i=1}^{T} n_i$. Similarly, we proceed with $\beta_{2+}$ and $\beta_{2-}$, but using the appropriate sample sizes $N^{+}_J = \sum_{i=1}^{T} n_i 1\{z_{2,i}>0\}$ and $N^{-}_J = N_J - N^{+}_J$ and with $\pi_{\beta_{2+}}(\beta_{2+}) \sim \text{invGam}(\delta_{\beta_{2+}1}, \delta_{\beta_{2+}2})$ and $\pi_{\beta_{2-}}(\beta_{2-}) \sim \text{invGam}(\delta_{\beta_{2-1}}, \delta_{\beta_{2-2}})$ being the corresponding priors.

Conditional on $X$, $\beta_{1+}^p$, $\beta_{2+}^p$, $\beta_{2-}^p$, $q_2$, $\Theta_M$ and jump times and sizes, using the jump adjusted processes $\tilde{Y}_i = Y_i - j_i n_i$, $\tilde{X}_{1i} = X_{1i} - z_{1,i} n_i$ and $\tilde{X}_{2i} = X_{2i} - z_{2,i} n_i$, we can sample $\mu^p$, $\kappa_{11}^p$, $\kappa_{22}^p$, and $\kappa_{21}^p$ using the standard normal multivariate regression model with known variance. In this context, prior information for those parameters can be easily introduced through Gaussian conjugate priors. Finally, we use a Metropolis step to compute the conditional posterior of $\rho_1$ and $\rho_2$, which is proportional to $p(y_i, X_i | X_{i-1}, j_i, z_i, n_i)$. 

9
<table>
<thead>
<tr>
<th>Article</th>
<th>Data</th>
<th>Period</th>
<th>Frequency</th>
<th>Model</th>
<th>VRP 1M</th>
<th>VRP 2M</th>
<th>VRP 1Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aït-Sahalia et al. (2014)</td>
<td>VS</td>
<td>1996 - 2010</td>
<td>Daily</td>
<td>AJD</td>
<td>[-7%, 0%]</td>
<td>[-10%, -0.5%]</td>
<td></td>
</tr>
<tr>
<td>Amengual (2008)</td>
<td>VS</td>
<td>1996 - 2007</td>
<td>Daily</td>
<td>AJD</td>
<td>[-5%, 0%]</td>
<td>[-7%, -0.5%]</td>
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</tr>
<tr>
<td>Bollerslev et al. (2011)</td>
<td>SPO</td>
<td>1990 - 2003</td>
<td>Intraday</td>
<td>MF</td>
<td>[-20%, 5%]*</td>
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<td></td>
</tr>
<tr>
<td>Corradi et al. (2013)</td>
<td>VIX</td>
<td>1950 - 2006</td>
<td>Monthly</td>
<td>AD</td>
<td>[-30%, 8%]*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fan et al. (2016)</td>
<td>VIX</td>
<td>2006 - 2011</td>
<td>Intraday</td>
<td>MF</td>
<td>[-30%, 20%]*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fusari and Gonzalez-Perez (2012)</td>
<td>SPO</td>
<td>1996 - 2010</td>
<td>Daily</td>
<td>Log-OU</td>
<td>[-20%, 3%]</td>
<td></td>
<td>[-23%, 1%]</td>
</tr>
<tr>
<td>Zhou (2009)</td>
<td>VIX</td>
<td>1990 - 2008</td>
<td>Intraday</td>
<td>MF</td>
<td>[-200%, 40%]</td>
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<td></td>
</tr>
</tbody>
</table>

Table A.1: Selection of Literature Reporting Variance Risk Premia Estimates

Note: This table collects estimates from papers that report either the point estimates or time series plots of variance or volatility risk premia. In the Data column, “VS” denotes variance swaps, “SPO” denotes S&P 500 options, and “VIX” is the CBOE volatility index. In the Model column, “AD” refers to affine diffusion, “AJD” denotes affine-jump diffusion, “MF” means model-free, and “Log-OU” is a log-affine process with two Ornstein-Uhlenbeck factors. The columns of VRP provide the approximate bounds of the estimated time series of risk premia, with 1M, 3M and 1Y referring to the 1-month, 2-month, and 1-year time-to-maturities. Most MF methodologies provide positive estimates of variance risk premia for certain periods of time, so that their upper bounds are positive.

* provides volatility risk premia.

** gives a point estimate with the standard error provided in the brackets.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>$A_0(2)$ True</th>
<th>Bias</th>
<th>Stdev</th>
<th>HPD 95%</th>
<th>$A_1(2)$ True</th>
<th>Bias</th>
<th>Stdev</th>
<th>HPD 95%</th>
<th>$A_2(2)$ True</th>
<th>Bias</th>
<th>Stdev</th>
<th>HPD 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_0^0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
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<td>0.016</td>
<td>$[ -4.537, -4.467]$</td>
<td>$-4.500$</td>
<td>$0.008$</td>
<td>0.013</td>
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<td>$-4.500$</td>
<td>$0.002$</td>
<td>0.008</td>
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<td>$0.001$</td>
<td>0.014</td>
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<td>$-0.200$</td>
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<tr>
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<td>$[ -0.213, -0.187]$</td>
<td>$-0.200$</td>
<td>0.000</td>
<td>0.009</td>
<td>$[ -0.217, -0.178]$</td>
<td>$-0.250$</td>
<td>0.001</td>
<td>0.002</td>
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<td>$0.001$</td>
<td>0.005</td>
<td>$[ 0.138, 0.158]$</td>
<td>0.150</td>
<td>$0.001$</td>
<td>0.002</td>
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<tr>
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<td>0.002</td>
<td>0.004</td>
<td>$[ 0.035, 0.049]$</td>
<td>0.400</td>
<td>0.000</td>
<td>0.004</td>
<td>$[ 0.391, 0.407]$</td>
<td>0.300</td>
<td>0.000</td>
<td>0.002</td>
<td>$[ 0.296, 0.306]$</td>
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<tr>
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<td>0.005</td>
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<td>0.000</td>
<td>0.006</td>
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<td>0.001</td>
<td>0.002</td>
<td>$[ -0.013, -0.003]$</td>
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<td>0.002</td>
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<tr>
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<td>0.640</td>
<td>1.087</td>
<td>$[ 1.928, 6.043]$</td>
<td>20.000</td>
<td>0.152</td>
<td>1.218</td>
<td>$[ 18.011, 23.126]$</td>
<td>20.000</td>
<td>$-0.047$</td>
<td>1.218</td>
<td>$[ 16.879, 22.296]$</td>
</tr>
<tr>
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<td>12.000</td>
<td>$-1.940$</td>
<td>2.758</td>
<td>$[ 1.873, 14.983]$</td>
<td>$-20.000$</td>
<td>$-0.165$</td>
<td>2.293</td>
<td>$[ -26.405, -16.228]$</td>
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<td>0.630</td>
<td>1.787</td>
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<tr>
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<td>$-0.048$</td>
<td>1.397</td>
<td>$[ -1.490, 3.791]$</td>
<td>$2.000$</td>
<td>$-0.589$</td>
<td>1.525</td>
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<td>$-12.000$</td>
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<td>1.567</td>
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<td>0.021</td>
<td>0.854</td>
<td>$[ 0.541, 13.215]$</td>
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<td>$-0.142$</td>
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<tr>
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<td>$-0.161$</td>
<td>0.395</td>
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<td>0.000</td>
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<td>0.000</td>
<td>$[ -3.500, -3.500]$</td>
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<td>0.000</td>
<td>0.000</td>
<td>$[ -3.850, -3.850]$</td>
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<td>0.000</td>
<td>$[ -6.400, -6.400]$</td>
</tr>
<tr>
<td>$\Pi_{11}$</td>
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<td>0.000</td>
<td>0.000</td>
<td>$[ 1.500, 1.500]$</td>
<td>1.250</td>
<td>0.000</td>
<td>0.000</td>
<td>$[ 1.250, 1.250]$</td>
<td>1.250</td>
<td>0.000</td>
<td>0.000</td>
<td>$[ 1.250, 1.250]$</td>
</tr>
<tr>
<td>$\Pi_{42}$</td>
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<td>0.000</td>
<td>$[ 0.500, 0.500]$</td>
<td>0.400</td>
<td>0.000</td>
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<td>0.400</td>
<td>0.000</td>
<td>0.000</td>
<td>$[ 0.400, 0.400]$</td>
</tr>
</tbody>
</table>

### Table A.2: Simulation Results for $\Theta_M$ and $\Theta_H$

Note: This table provides a summary of a Monte Carlo simulation exercise with 100 replications for the two examples of two-factor volatility models that we introduce in Appendix ???. We report true values, bias and standard deviations across the simulations for $\Theta_M$, the parameters determining the dynamics of the latent factors under the risk-neutral measure, and $\Theta_H$, the parameters defining $f$. 
<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\kappa_{11}$</th>
<th>$\kappa_{12}$</th>
<th>$\nu$</th>
<th>$\xi$</th>
<th>$\beta_{1+}$</th>
<th>$\beta_{1-}$</th>
<th>$\beta_{2+}$</th>
<th>$\beta_{2-}$</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\mu^p_1$</th>
<th>$\mu^p_2$</th>
<th>$s_{2M}$</th>
<th>$s_{2Y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>-1.750</td>
<td>0.200</td>
<td>-3.600</td>
<td>4.800</td>
<td>-0.600</td>
<td>0.200</td>
<td>0.180</td>
<td>0.200</td>
<td>0.200</td>
<td>0.010</td>
<td>-0.700</td>
<td>-0.450</td>
<td>0.070</td>
<td>0.150</td>
<td>0.050</td>
<td>0.050</td>
</tr>
<tr>
<td>Bias</td>
<td>-0.264</td>
<td>-0.315</td>
<td>-0.260</td>
<td>0.011</td>
<td>-0.093</td>
<td>-0.002</td>
<td>0.009</td>
<td>0.008</td>
<td>0.005</td>
<td>0.000</td>
<td>0.017</td>
<td>0.018</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>Stdev</td>
<td>0.694</td>
<td>0.938</td>
<td>0.789</td>
<td>0.877</td>
<td>0.222</td>
<td>0.044</td>
<td>0.041</td>
<td>0.044</td>
<td>0.032</td>
<td>0.002</td>
<td>0.041</td>
<td>0.040</td>
<td>0.046</td>
<td>0.007</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>HPD 95%</td>
<td>[-3.518, -0.903]</td>
<td>[-2.203, 1.503]</td>
<td>[-5.653, -2.547]</td>
<td>[2.965, 6.451]</td>
<td>[-1.185, -0.324]</td>
<td>[0.119, 0.293]</td>
<td>[0.122, 0.284]</td>
<td>[0.136, 0.309]</td>
<td>[0.148, 0.274]</td>
<td>[0.006, 0.015]</td>
<td>[-0.745, -0.578]</td>
<td>[-0.510, -0.354]</td>
<td>[-0.013, 0.169]</td>
<td>[0.138, 0.165]</td>
<td>[0.047, 0.055]</td>
<td>[0.047, 0.055]</td>
</tr>
</tbody>
</table>

### Table A.3: Simulation Results for $\Theta_P$ and $\Theta_E$

Note: This table provides a summary of a Monte Carlo simulation exercise with 100 replications for the two examples of two-factor volatility models that we introduce in Appendix ???. We report true values, bias and standard deviations across the simulations for the additional parameters that characterize the $\mathbb{F}$-dynamics, $\Theta_P$, and the pricing error variances, summarized in $\Theta_E$. 


<table>
<thead>
<tr>
<th>Indicator</th>
<th>Category</th>
<th>Frequency</th>
<th>Release Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment Rate</td>
<td>Employment</td>
<td>Monthly</td>
<td>8:30 am First Friday of each month</td>
</tr>
<tr>
<td>ADP Employment Change</td>
<td>Employment</td>
<td>Monthly</td>
<td>8:15 am - Two days before Employment situation</td>
</tr>
<tr>
<td>Initial Jobless Claims</td>
<td>Employment</td>
<td>Weekly</td>
<td>8:30 am every Thursday</td>
</tr>
<tr>
<td>Personal Income</td>
<td>Consumer Spending and Confidence</td>
<td>Monthly</td>
<td>8:30 am 4 weeks after end of reported month</td>
</tr>
<tr>
<td>Personal Spending</td>
<td>Consumer Spending and Confidence</td>
<td>Monthly</td>
<td>8:30 am 4 weeks after end of reported month</td>
</tr>
<tr>
<td>Advance Retail Sales</td>
<td>Consumer Spending and Confidence</td>
<td>Monthly</td>
<td>8:30 am 2 weeks after end of reported month</td>
</tr>
<tr>
<td>Consumer Confidence</td>
<td>Consumer Spending and Confidence</td>
<td>Monthly</td>
<td>10:00 am - Last Tuesday of month being surveyed</td>
</tr>
<tr>
<td>GDP</td>
<td>National Output and Inventories</td>
<td>Quarterly</td>
<td>8:30 am - Final week of Jan Apr Jul Oct</td>
</tr>
<tr>
<td>Durable Goods Orders</td>
<td>National Output and Inventories</td>
<td>Monthly</td>
<td>8:30 am three to four weeks after the end of reporting month</td>
</tr>
<tr>
<td>ISM Manufacturing</td>
<td>National Output and Inventories</td>
<td>Monthly</td>
<td>10:00 am First Business day after reporting month</td>
</tr>
<tr>
<td>Chicago PMI</td>
<td>National Output and Inventories</td>
<td>Monthly</td>
<td>10:00 am First Business day of month being covered</td>
</tr>
<tr>
<td>Empire State Manufacturing</td>
<td>National Output and Inventories</td>
<td>Monthly</td>
<td>8:30 am around 15th of month being reported</td>
</tr>
<tr>
<td>Business Inventories</td>
<td>National Output and Inventories</td>
<td>Monthly</td>
<td>10:00 am released six weeks after the month ends</td>
</tr>
<tr>
<td>Production and Utilization</td>
<td>National Output and Inventories</td>
<td>Monthly</td>
<td>9:15 am released the 15th of the following month</td>
</tr>
<tr>
<td>New Residential Sales</td>
<td>Housing and Construction</td>
<td>Monthly</td>
<td>8:30 am released two to three weeks following month being covered</td>
</tr>
<tr>
<td>FOMC Meetings</td>
<td>Central Bank</td>
<td>Eight Times</td>
<td>2:15 pm of day of conclusion of FOMC meetings</td>
</tr>
<tr>
<td>Fed Chairman’s Speeches</td>
<td>Central Bank</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>ECB Meetings</td>
<td>Central Bank</td>
<td>Monthly</td>
<td>N.A.</td>
</tr>
<tr>
<td>CPI</td>
<td>Prices, Productivity, Wages</td>
<td>Monthly</td>
<td>8:30 am second or third week following month being covered</td>
</tr>
<tr>
<td>PPI</td>
<td>Prices, Productivity, Wages</td>
<td>Monthly</td>
<td>8:30 am two or three weeks after month ends</td>
</tr>
<tr>
<td>Employment Cost Index</td>
<td>Prices, Productivity, Wages</td>
<td>Quarterly</td>
<td>8:30 am - Last Thursday of Jan Apr Jul Oct</td>
</tr>
</tbody>
</table>

**Table A.4: Economic Indicators**

Note: In this table, we report the details of the 21 macroeconomic news announcements or central bank events used in Section 4.1. All times are reported in Eastern Standard Time. Source: Bloomberg and the book by Baumohl (2010) on economic indicators.
Figure A.1: Volatility Factors

Note: This figure reports the posterior means (in blue) of the two latent factor estimates for $A_0(2)$, $A_1(2)$, $A_2(2)$, $A_0^+(2)$, and $A_1^+(2)$, as well as the logarithm of the factors for the $A_2(2)$ model. The red areas around the blue curves mark the 95%-credible sets. We use daily data on variance swaps from January 4, 1996 to January 11, 2013. The number of daily observations is 4,276, excluding weekends and holidays.
Figure A.2: Out of Sample Performance

Note: This figure compares the estimated 1-month variance swap rates with the VIX across models over the entire sample period. The red solid line denotes the VIX from the CBOE, whereas the blue dash-dotted line is calculated based on the $Q$-parameters estimated from the variance swap rates with time-to-maturity of at least 2 months.
References


Baumohl, B. (2010), *The Secrets of Economic Indicators*, Prentice Hall.


