Inference on Risk Premia in the Presence of Omitted Factors

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Introduction

- Theoretical asset pricing models predict that *some* factors are priced
- Such models are typically very stylized, with 1-2 factors, e.g., CAPM, hence rejected by the data
- Plausible that the literal model is missing some factors (Ross' APT) or the factor of interest is measured with error (Roll's critique)
- Can we address **whether the factor predicted by a theoretical model is priced**, without specifying all factors in the model, while allowing for measurement error?

Introduction

- Focus on the **risk premium** of a factor, instead of testing the entire model
  - *How much are investors willing to pay to hedge that risk (and just that risk)?*
- Easy to do for **traded** factors
  - avg excess return of the portfolio
  - no need to specify a full-fledged model
- **Nontraded** factors: several ways
  1. Multidimensional portfolio sorting
  2. Two-pass Fama-MacBeth (FM), GMM

Introduction

Either method is **biased** when there are omitted factors, or the factor of interest is measured with error.

- Portfolios with high market betas have smaller excess returns than those with low market betas.
- FM estimate of market risk premium is negative and significant $\neq$ the average market return.
- Controlling Fama-French factors does not solve this problem.

Toy Example I - Measurement Error

Consider a one factor model (e.g., CAPM):

$$r_t = \beta\gamma + \beta v_t + u_t, \quad g_t = v_t + z_t, \quad \mathrm{E}(z_t|v_t) = 0$$

If using the standard FM regression with g_t:

- $\widehat{\beta}$ is biased towards 0 (attenuation bias)
- $\widehat{\gamma}$ is biased upwards
- the smaller the signal-to-noise ratio, the worse the bias.

Toy Example II - Missing Factors

Consider a two factor model:

$$r_t = \beta_1\gamma_1 + \beta_2\gamma_2 + \beta_1 v_{1t} + \beta_2 v_{2t} + u_t.$$

How to estimate γ_1 if we only observe $g_t = v_{1t}$ and miss v_{2t}?

- even if $v_{1t} \perp v_{2t}$, it is not enough to consistently estimate γ_1
- orthogonality between factors is enough to guarantee consistent estimate of β_1
- recovery of γ_1 requires an additional constraint $\beta_1 \perp \beta_2$!

$$\mathrm{E}(r_t) = \beta_1\gamma_1 + \beta_2\gamma_2$$

This paper

This paper proposes a **three-pass** methodology to address the omitted factor and measurement error problems in the two-pass FM regressions

- Rotation invariance result
  - Risk premia estimate for g_t is the same for any rotation of the controls as long as they together span the entire factor space
  - No need to know the **identities** of the omitted factors
- Recover the factor space via PCA from a **large** panel of returns
  - Say something about the **economically interpretable** g_t, using latent factors from returns only as *controls*.
  - Exploit the benefits of factor analysis but address its well-known uninterpretability (APT)

Finance Literature

- Model misspecification and weak identification

  Kan and Zhang (1999a,b), Shanken and Zhou (2007), Kan and Robotti (2008), Kleibergen (2009), Kan and Robotti (2009), Lewellen et al. (2010), Kan et al. (2013), Gospodinov et al. (2013), Kleibergen and Zhan (2014), Gospodinov et al. (2014b), Bryzgalova (2015)

  - In particular, there is a weak factor v_{2t} included, for which $\beta_2 \approx 0$.

    $$r_t = \beta_1\gamma_1 + \beta_2\gamma_2 + \beta_1 v_{1t} + \beta_2 v_{2t} + u_t.$$

    $\widehat{\beta}_2$ would be close to 0, which would then mess up γ_1.
  - In contrast, we mainly focus on the case where v_{2t} should be added ($\beta_2 \neq 0$) but is missing. (NEW)

- Other related work mainly focus on mispricing, i.e., "α"

  Connor and Korajczyk (1986, 1988), Fan, Liao, and Yao (2016), etc

Econometrics Literature

- (Approximate) Factor model

  Ross (1976), Chamberlain and Rothchild (1983)

- Determine the number of factors

  Bai and Ng (2002), Onatski (2010), Ahn and Horenstein (2013)

- Inference, Forecasting, etc

  Stock and Watson (2002), Bai and Ng (2006, 2008), Bai (2003, 2009), Moon and Weidner (2015), Fan, Liao, and Mincheva (2011, 2013).

Model Setup

- True model is linear with p factors v_t (zero-mean).

$$r_t = \alpha + \iota_n \gamma_0 + \beta\gamma + \beta v_t + u_t$$

- γ are the factor risk premia
- β are the risk exposures
- Factor of interest g_t (tradable or nontradable):

$$g_t = \xi + \eta v_t + z_t$$

- z_t is measurement error, that we will also account for

Risk premium of g_t is the expected excess return of a "pure factor" portfolio with beta of 1 with g_t (free of measurement error) and 0 with all other risk sources. For g_t, it is $\eta\gamma$.

Rotation invariance: an example

- Standard FM using the 25 FF portfolios

Slope	RmRf	SMB	HML
Estimate	-112	12	40
Stderr	(44)	(10)	(11)

- Construct two new factors:

$$F_2 = \mathbf{a} * RmRf + \mathbf{b} * SMB + \mathbf{c} * HML$$

$$F_3 = RmRf + 0.001 HML$$

- **What happens if we run FM using** $RmRf, F_2, F_3$?
  - All factors are contaminated by $RmRf$: **betas** will be wrong
  - **a**, **b**, **c** are unknown constants
  - F_3 is almost collinear with $RmRf$

Rotation invariance: an example

- Standard Fama-MacBeth using the 25 FF portfolios

Slope	RmRf	SMB	HML
Estimate	**-112**	12	40
Stderr	**(44)**	(10)	(11)

- Rotated model:

Slope	RmRf	F_2	F_3
Estimate	**-112**	-100	-1116
Stderr	**(44)**	(47)	(444)

- The slope of g_t in any **FM regression** is $\eta\gamma$, as long as g_t, together with any linear combinations of v_t, span the true factor space.
- Multicolinearity in these regressions is not a problem.

Rotation Invariance Result

- In the case when v_t is latent and g_t is measured with error (focus of this paper): it is natural to use PCA to recover the factor space, and use an additional regression on latent factors to de-noise

$$r_t = \alpha + \iota_n \gamma_0 + \beta H^{-1} H \gamma + \beta H^{-1} H v_t + u_t$$

$$g_t = \xi + \eta H^{-1} H v_t + z_t$$

  The risk premium of g_t in the rotated model is $\eta H^{-1} H \gamma = \eta \gamma$, regardless of the rotation H.

- Risk exposure (β) is not rotation-invariant.

A Three-pass Estimator: Invariance Result + PCA

We propose a **three-pass estimator** to obtain $\eta\gamma$.

1. Extract latent factors $\widehat{v}_t$ via PC
2. Use cross-sectional regression to estimate latent factor risk premia $\hat{\gamma}$
3. Regress g_t on the latent factors $\widehat{v}_t$ via time-series regression to obtain $\hat{\eta}$

Risk premium of g_t is estimated as $\hat{\eta}\hat{\gamma}$

Another View

Risk premium of g_t is $-\gamma_0 Cov(g_t, m_t)$, where m is the SDF

- Steps 1 and 2 recover the SDF (under APT):

$$m_t = \gamma_0^{-1}(1 - \gamma^\intercal \Sigma_v^{-1} v_t)$$

- Step 3 computes risk premia as $-\gamma_0 Cov(g_t, m_t)$

Invariance holds because SDF is rotation-invariant and the risk premium depends on a univariate covariance

Risk Premia vs Risk Prices

1. What is the **risk premium** of g_t?
   - *How much are investors willing to pay to hedge that risk?*
2. What is the **risk price** of g_t?
   - *Does g_t help price the cross-section of assets?*
3. They are related:

$$\Sigma_v^{-1}\gamma = \lambda$$
$$m_t = \gamma_0^{-1}(1 - \gamma^\intercal \Sigma_v^{-1} v_t) = \gamma_0^{-1}(1 - \lambda^\intercal v_t)$$

- The invariance result does not hold for risk prices
- Risk prices depend on the covariance matrix of the other factors Σ_v

Asymptotic Theory

- Estimator is consistent and satisfies CLT as $T, n \to \infty$. Convergence rate is $\min(n^{1/2}, T^{1/2})$.
- Derive asymptotic distribution under fairly general assumptions
- Heteroscedasticity robust standard errors
- The usual GLS is not necessarily more efficient than OLS so that we can avoid large-dim covariance estimates (Σ^u) in our inference, because it appears at $O_p(n^{-1/2}T^{-1/2}) << O_p(n^{-1/2} + T^{-1/2})$.
- Robust even if a few extra PCs are used.
- Also provide inference for $\widehat{g}_t$, which requires $\widehat{\Sigma}^u$.
- Also provide a Wald test of $\mathbb{H}_0 : \eta = 0$ (i.e., g_t is weak).

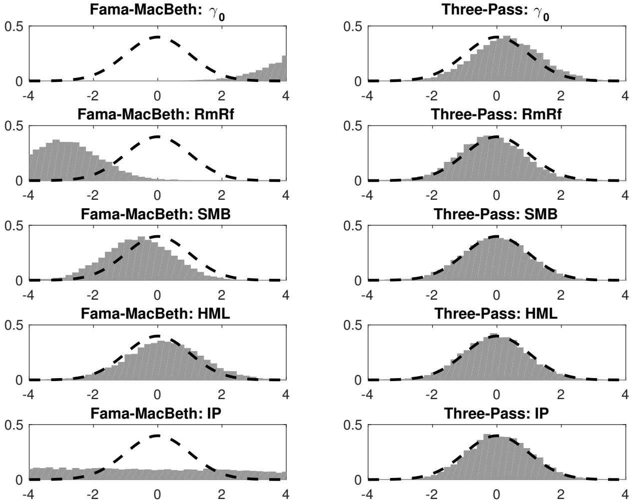
A Critical Assumption

- All latent factors are pervasive (strong):

$$\|n^{-1}\beta^{\intercal}\beta - \Sigma^{\beta}\| = o(1), \quad \text{as} \quad n \to \infty$$

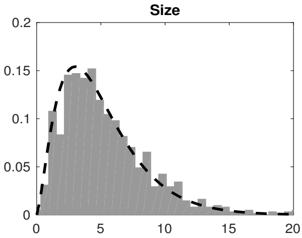
- Our view of the weak factor literature:
  The true (latent) fundamental economic factors are strong, but we do not have good (observable) proxies for such factors.
- For example, macro factors in the literature are weak, see Gospodinov et al. (2014b), Bryzgalova (2015), but stock market strongly reacts to Fed and Government policies, see Bernanke and Kuttner (2005), Pastor and Veronesi (2012, 2014).

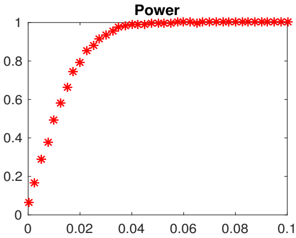
Simulation: Estimation



Simulation: Testing

$$\mathbb{H}_0 : \eta = 0, \quad \text{i.e., factor } g_t \text{ is weak}$$





Testing portfolios and their factor structure

- Test assets: 202 standard equity portfolio from Fama and French
  - 25 portfolios sorted by size and book-to-market
  - 17 industry portfolios
  - 25 portfolios sorted by operating profitability and investment
  - 25 portfolios sorted by size and variance
  - 35 portfolios sorted by size and net issuance
  - 25 portfolios sorted by size and accruals
  - 25 portfolios sorted by size and momentum
  - 25 portfolios sorted by size and beta.
- We first need to select the number of PCs
  - 4 PCs have average time series R^2 of 65%
  - Robustness to increasing the number of PCs (4,5,6)

Portfolios vs. individual assets

- We use **portfolios** throughout, as our theory assumes constant betas
  - Individual asset betas are unstable
  - If betas are a function of characteristics, characteristics-sorted portfolios have constant betas

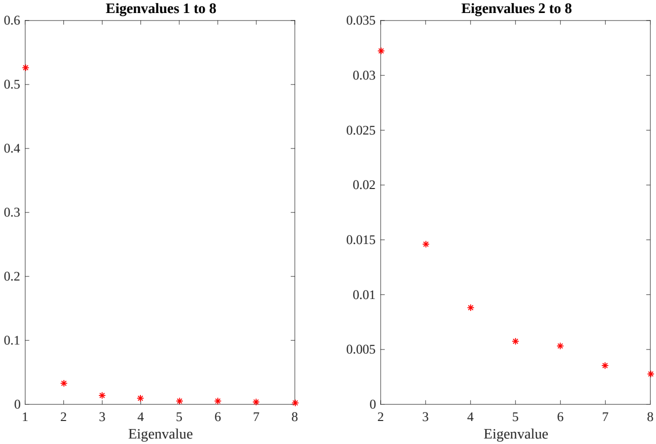
For instance, if $\tilde{r}_{t+1}$ is excess-return vector of individual stocks with time-varying β_t explained by observable characteristics:

$$\tilde{r}_{t+1} = \beta_t \gamma + \beta_t v_{t+1} + \tilde{u}_{t+1}, \quad \text{and} \quad \beta_t = c_t \beta$$

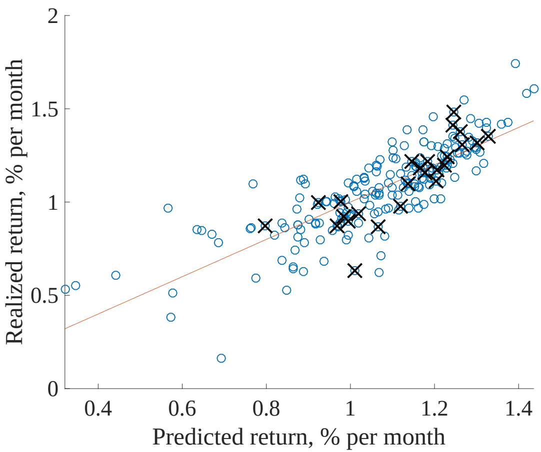
then

$$r_{t+1} := (c_t^\intercal c_t)^{-1} c_t^\intercal \tilde{r}_{t+1} = \beta\gamma + \beta v_{t+1} + (c_t^\intercal c_t)^{-1} \tilde{u}_{t+1}.$$

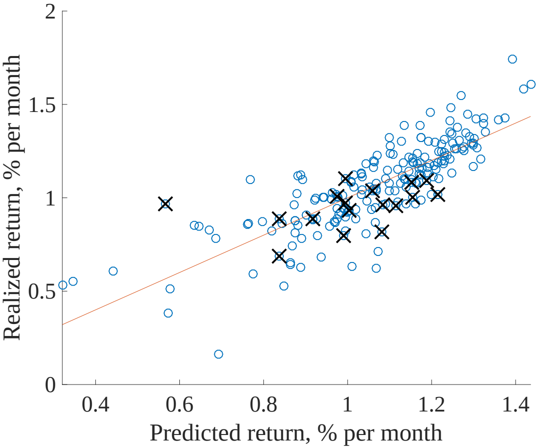
Factor selection



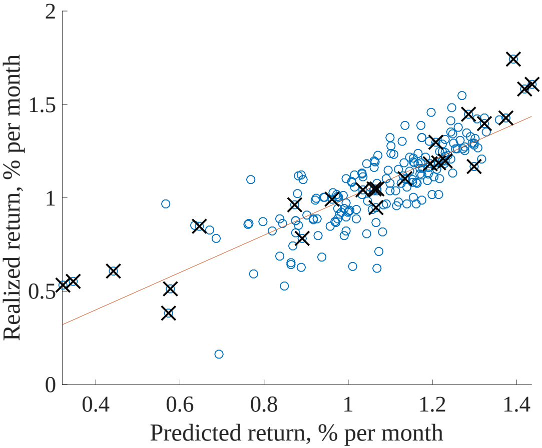
PC fit: BE/BE-ME



PC fit: Industries



PC fit: ME/Momentum



Observable factors

- Consider many tradable and nontradable factors
  - CAPM: Rm
  - Fama-French 3-factor model (FF3): Rm, SMB, HML
  - Carhart 4-factor model (FF4): Rm, SMB, HML, Momentum
  - Fama-French 5-factor model (FF5): Rm, SMB, HML, CMA (investment), RMW (profitability)
  - Q-factor model (HXZ): Rm, ME (size), IA (investment), ROE (profitability)
  - Betting against beta (BAB)
  - Quality-minus-junk (QMJ)
  - Industrial production growth (IP)
  - Jurado, Ludvigson and Ng macro factors (LN)
  - Liquidity factor from Pastor and Stambaugh 2003 (LIQ)
  - Intermediary factors from He et al. (2016) and Adrian et al. (2015)

Results: Tradable factors, Fama-MacBeth vs. 3-pass

			FM			3-pass, $P = 4$			
Model	Factors	Avg ret	γ		stderr	γ		stderr	R_g^2
FF3	RmRf	0.50	-0.57	**	(0.25)	0.37	*	(0.20)	98.18
	SMB	0.23	0.17		(0.13)	0.23	*	(0.13)	93.90
	HML	0.34	0.23	*	(0.13)	0.21	*	(0.11)	66.86

- Robust to using P=4, 5, 6
- These are **tradable** factors, and they are not weak.
- If the model is correctly specified, we would expect the risk premia to be equal to the average excess returns

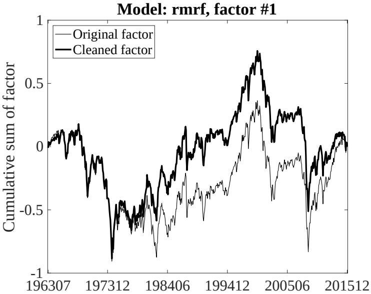
Results: Nontradable factors

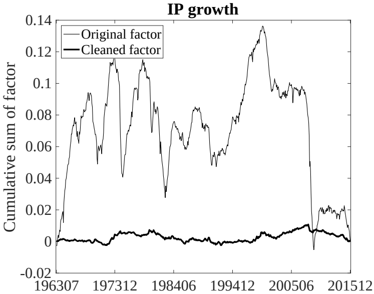
	FM			3-pass, $P = 4$				
Factors	γ		stderr	γ		stderr	R^2_g	p-vaule
IP	-0.13	*	(0.07)	-0.00		(0.00)	0.13	0.96
LN 1	70.25	***	(21.62)	1.74		(1.32)	0.60	0.47
LN 2	3.84		(24.02)	-2.25		(1.89)	2.57	0.04
LN 3	-1.71		(15.04)	1.09		(1.78)	3.58	0.03
Liquidity	0.02		(0.97)	0.26	**	(0.12)	11.99	0.00
He et al.	0.02		(0.64)	0.30		(0.27)	60.10	0.00
Adrian et al.	1.25	***	(0.32)	0.79	***	(0.16)	49.28	0.00

Factor Loadings

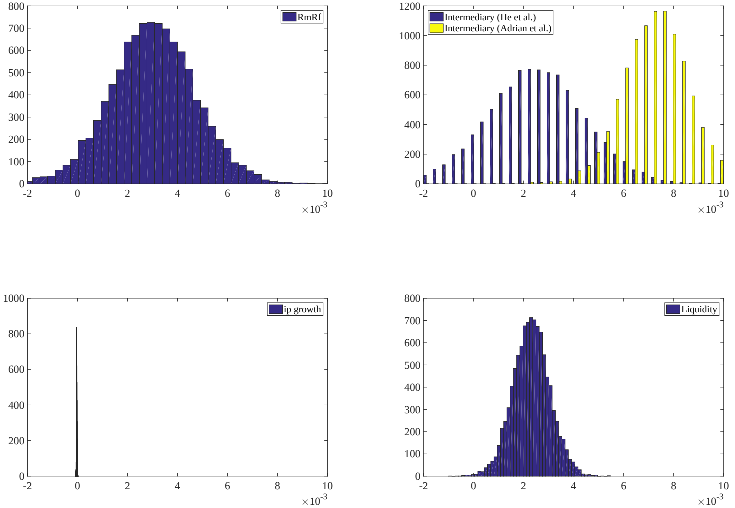
Model	Factors	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
CAPM	RmRf	91.0	6.3	1.7	0.1	0.8	0.2
FF3	SMB	31.0	64.0	0.6	0.9	1.0	2.4
	HML	7.0	1.3	75.5	4.9	1.4	9.9
IP	IP Growth	2.2	0.5	4.4	1.0	3.8	88.0
Liq.	Liquidity	95.0	2.8	1.8	0.2	0.2	0.0
Interm.	He et al.	81.2	12.5	0.1	2.9	3.2	0.1
	Adrian et al.	20.3	6.8	52.0	16.4	0.0	4.5

Measurement Error

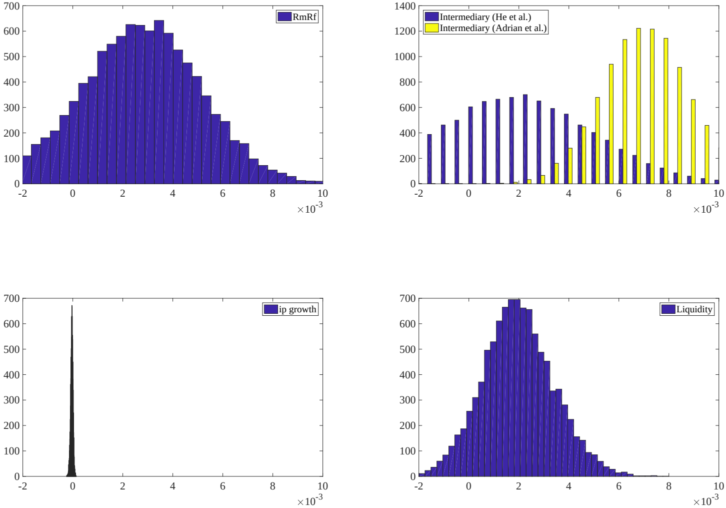




Robustness to Choice of Testing Portfolios



Robustness Across Time Periods



Risk Premia across Asset Classes

		202 equity		FF25, 100 non-eq.		100 non-equity	
Model	Factors	γ	stderr	γ	stderr	γ	stderr
FF3	RmRf	0.29	(0.25)	0.73***	(0.23)	0.65***	(0.22)
	SMB	0.18	(0.14)	0.22*	(0.12)	0.13**	(0.06)
	HML	0.25*	(0.13)	0.16**	(0.07)	0.08	(0.07)
IP	IP	−0.00	(0.01)	0.01	(0.01)	0.01	(0.01)
Liq.	Liq.	0.22	(0.14)	0.24	(0.18)	0.19	(0.17)
Interm.	He	0.28	(0.30)	1.03***	(0.30)	0.95***	(0.30)
	Adrian	0.77***	(0.17)	0.50***	(0.11)	0.35***	(0.09)

Conclusion

- Marry PCA with invariance result to correct for omitted variable bias
- Two caveats:
  1. PCA might miss latent weak factors
     - Tradable factor results
     - Robustness to the number of factors
     - Good deal bounds
  2. Invariance result does not hold for the risk price of g_t
     - Use tools from machine learning/model selection to recover the omitted factors and control for them (Feng, Giglio and Xiu 2017).
- Main point: when computing risk premia, literature typically use few arbitrary factors as controls. Crucial to to **complete the space**.